Tighter DFR Analysis & New Decoders for HQC

Marco Baldi, Sebastian Bitzer, Nicholas Lilla, Paolo Santini

Technical University of Munich
Università Politecnica delle Marche

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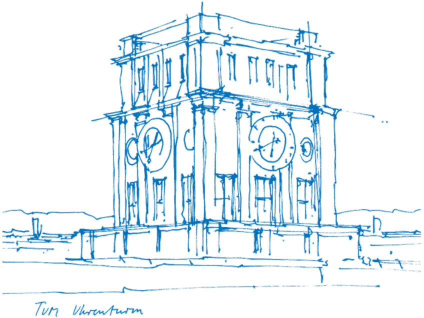

Hamming Quasi-Cyclic (HQC)

- Aguilar-Melchor, C., et al. (2017). Hamming quasi-cyclic (HQC). *NIST PQC*
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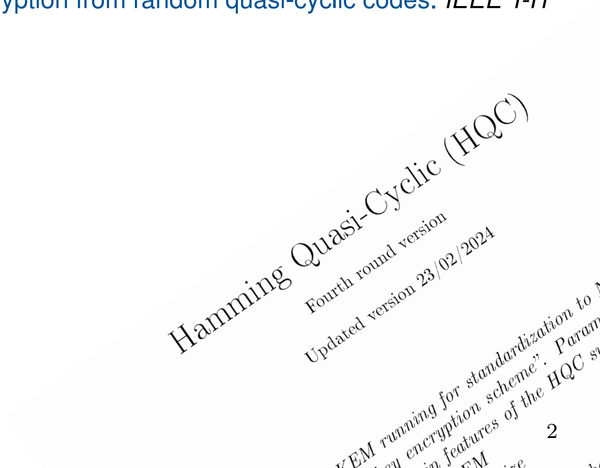
Hamming Quasi-Cyclic (HQC)

Fourth round version

Updated version 23/02/2024

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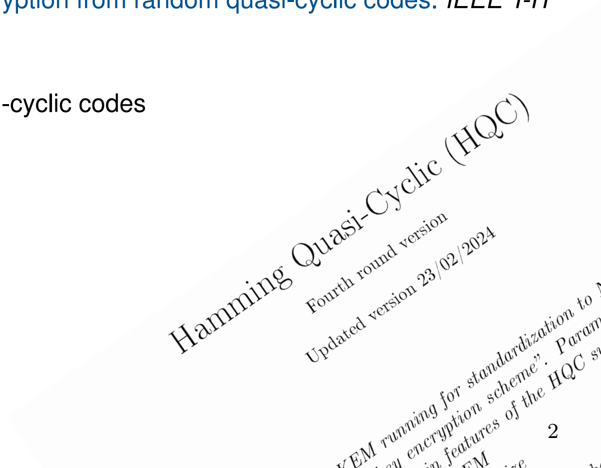
- Based on hardness of decoding random quasi-cyclic codes
- No hidden code structure
- Precise DFR analysis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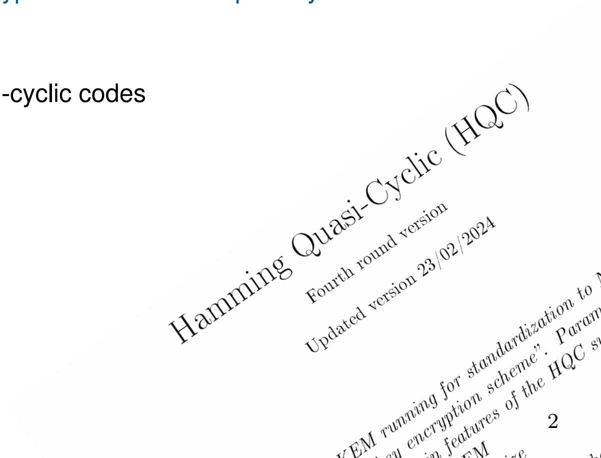

HQC in a Nutshell

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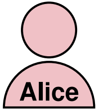
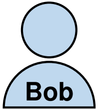




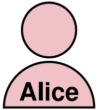
$\boldsymbol{h} \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$

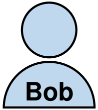
$\boldsymbol{u}_1, \boldsymbol{u}_2 \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$ of wt w_u

$\boldsymbol{s} \leftarrow \boldsymbol{u}_1 + \boldsymbol{h}\boldsymbol{u}_2$

$$\xrightarrow{(\boldsymbol{h}, \boldsymbol{s})}$$

HQC in a Nutshell





Alice:

$\boldsymbol{h} \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$

$\boldsymbol{u}_1, \boldsymbol{u}_2 \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$ of wt w_u

$\boldsymbol{s} \leftarrow \boldsymbol{u}_1 + \boldsymbol{h}\boldsymbol{u}_2$

$\xrightarrow{(\boldsymbol{h}, \boldsymbol{s})}$

Bob:

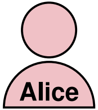
$\boldsymbol{c} \leftarrow \mathcal{C}.\textsc{Enc}(\boldsymbol{m})$

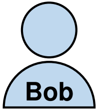
$\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$ of wt w_r

$(\boldsymbol{t}_1, \boldsymbol{t}_2) \leftarrow (\boldsymbol{c} + \boldsymbol{s}\boldsymbol{r}_2 + \boldsymbol{r}_3, \boldsymbol{r}_1 + \boldsymbol{h}\boldsymbol{r}_2)$

$\xleftarrow{(\boldsymbol{t}_1, \boldsymbol{t}_2)}$

HQC in a Nutshell





Alice		Bob
$\boldsymbol{h} \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$		
$\boldsymbol{u}_1, \boldsymbol{u}_2 \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$ of wt w_u		
$\boldsymbol{s} \leftarrow \boldsymbol{u}_1 + \boldsymbol{h}\boldsymbol{u}_2$	$\xrightarrow{(\boldsymbol{h}, \boldsymbol{s})}$	$\boldsymbol{c} \leftarrow \mathcal{C}.\textsc{Enc}(\boldsymbol{m})$
		$\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \xleftarrow{\$} \mathbb{F}_2[x]/(x^n - 1)$ of wt w_r
$\hat{\boldsymbol{m}} \leftarrow \mathcal{C}.\textsc{Dec}(\boldsymbol{t}_1 - \boldsymbol{t}_2\boldsymbol{u}_2)$	$\xleftarrow{(\boldsymbol{t}_1, \boldsymbol{t}_2)}$	$(\boldsymbol{t}_1, \boldsymbol{t}_2) \leftarrow (\boldsymbol{c} + \boldsymbol{s}\boldsymbol{r}_2 + \boldsymbol{r}_3, \boldsymbol{r}_1 + \boldsymbol{h}\boldsymbol{r}_2)$

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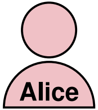

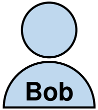

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$\xleftarrow{(\boldsymbol{t}_1, \boldsymbol{t}_2)}$

Alice:

$\hat{\boldsymbol{m}} \leftarrow \mathcal{C}.\textsf{Dec}(\boldsymbol{t}_1 - \boldsymbol{t}_2\boldsymbol{u}_2)$

$\mathcal{C}$ needs to decode $\boldsymbol{t}_1 - \boldsymbol{t}_2\boldsymbol{u}_2 = \boldsymbol{c} + \underbrace{\boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3}_{\text{error } \boldsymbol{e}}$

A First Look at the Error

- $P(|\boldsymbol{e}| = w)$ difficult for $\boldsymbol{e} = \boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3$
- $\rho = P(e_i = 1)$ simple

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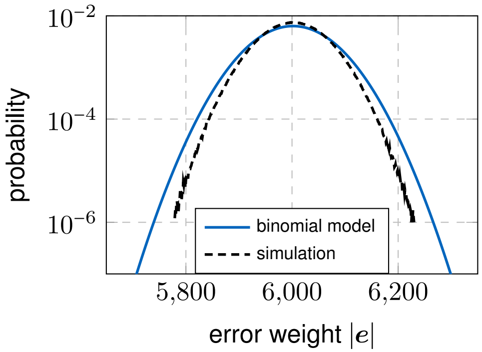
Binomial Approximation

Under the independence assumption,

$$P(|\boldsymbol{e}| = w) \approx \binom{n}{w}\rho^t(1-\rho)^{n-w}.$$



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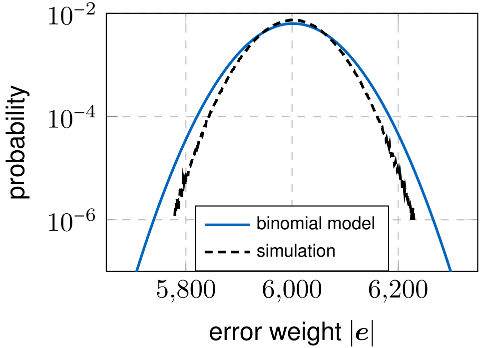
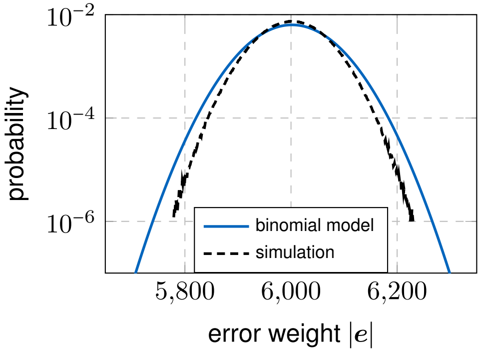


Seems conservative but not precise!

A Second Look at the Error

- Consider $\boldsymbol{a} = \boldsymbol{u} \cdot \boldsymbol{r} = \sum_{\ell \in \mathrm{supp}(\boldsymbol{u})} x^{\ell} \cdot \boldsymbol{r}(x)$



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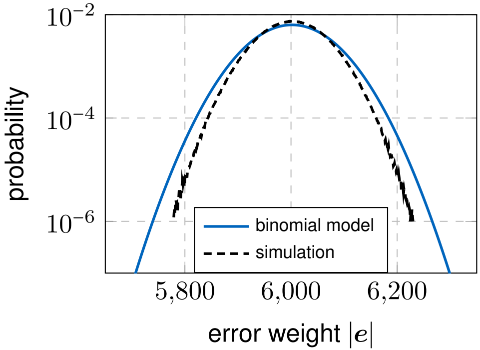

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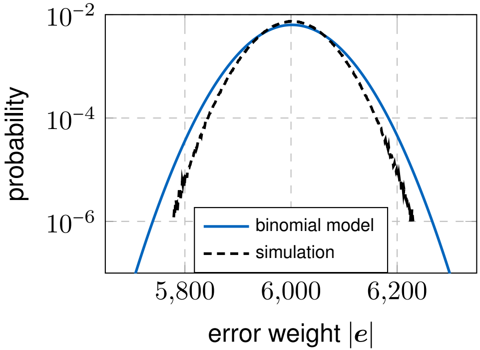
- $a_i = b_i \mod 2$
- $\sum_i b_i = |\boldsymbol{u}| \cdot |\boldsymbol{r}|$



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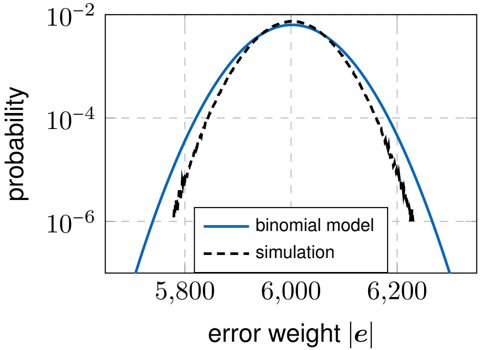
Proposed Approximation

Assume $b_0, \ldots, b_{n-1}$ indep. hypergeometric, let $a_i = b_i \mod 2$:

$$P(|\boldsymbol{u} \cdot \boldsymbol{r}| = w) \approx P\left(\sum_i a_i \mid \sum_i b_i = |\boldsymbol{u}| \cdot |\boldsymbol{r}|\right).$$



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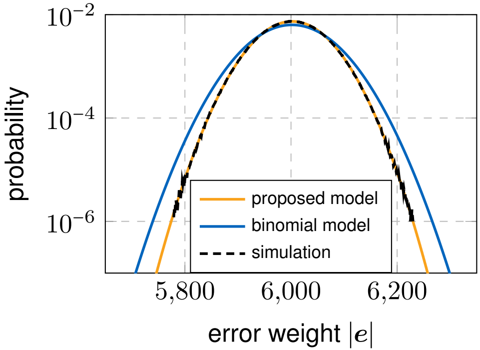
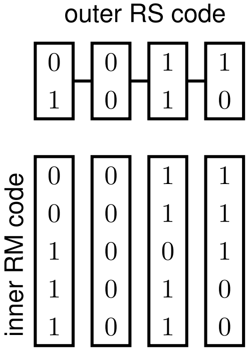


Impact on DFR estimation?

Tensor Product Code in HQC

Encoder

1. Encode outer RS code
2. Encode inner RM code



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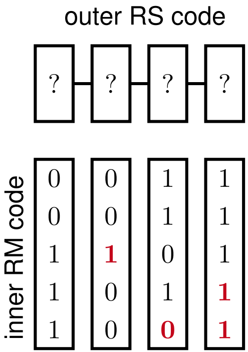
Tensor Product Code in HQC



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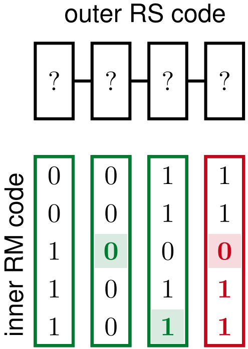

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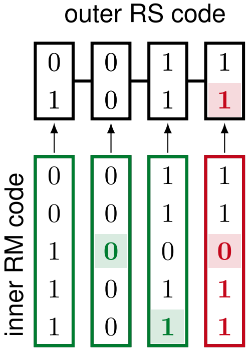

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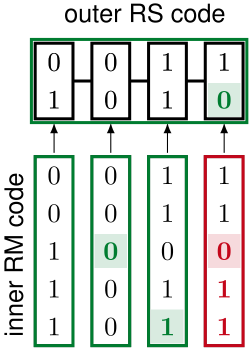

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Simple DFR analysis under independence assumption ✓

Tensor Product Code in HQC

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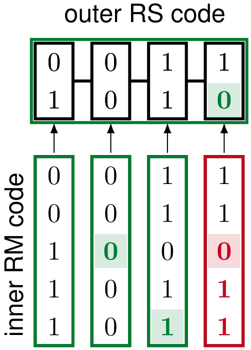


Decoder

1. Decode inner RM code
2. Decode outer RS code

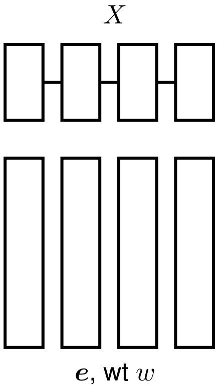
Simple DFR analysis under independence assumption ✓

But what about the proposed model?

DFR Analysis

Notation:

- X = # of erroneous outer symbols
- τ = correction capability outer code



DFR Analysis

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- X = # of erroneous outer symbols
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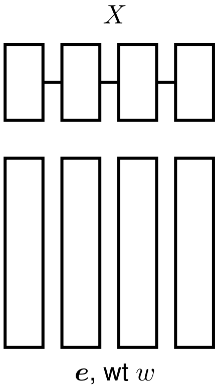
$\Rightarrow \mathrm{DFR} = \sum_w P(X > \tau \mid w) P(|\boldsymbol{e}| = w)$



DFR Analysis

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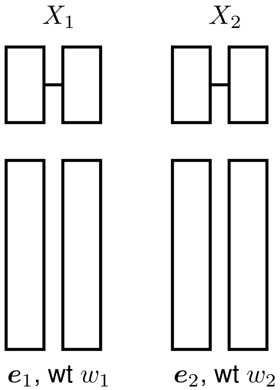
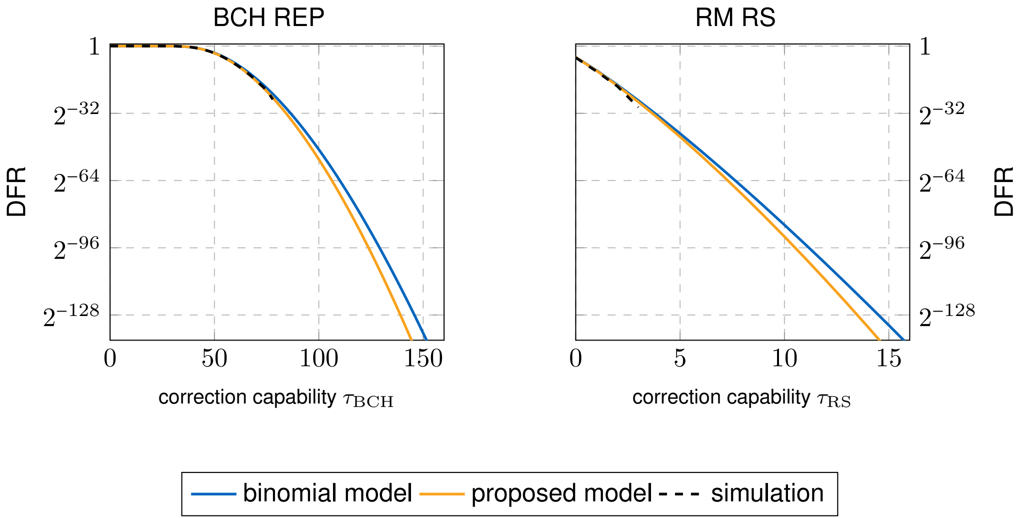
Divide and Analyze

Split $X = X_1 + X_2$ and $\boldsymbol{e} = (\boldsymbol{e}_1, \boldsymbol{e}_2)$
with $|\boldsymbol{e}_1| = w_1, |\boldsymbol{e}_2| = w - w_1$:

$$P(X \mid w) = \sum_{w_1} P(X_1 \mid w_1) * P(X_2 \mid w - w_1) \cdot P(w_1 \mid w)$$



DFR Comparison



Using the Error Structure for Decoding

Remember: $\boldsymbol{e} = \boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3$

$$\boxed{\begin{matrix}0\\0\\1\\1\\1\end{matrix}} \quad \boxed{\begin{matrix}0\\0\\\mathbf{1}\\0\\0\end{matrix}} \quad \boxed{\begin{matrix}1\\1\\0\\1\\\mathbf{0}\end{matrix}} \quad \boxed{\begin{matrix}1\\1\\1\\\mathbf{1}\\\mathbf{1}\end{matrix}}$$

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Proposed Decoder

1. Decode inner codewords

Using the Error Structure for Decoding

Remember: $\boldsymbol{e} = \boldsymbol{u}_1 \boldsymbol{r}_2 + \boldsymbol{u}_2 \boldsymbol{r}_1 + \boldsymbol{r}_3$

Proposed Decoder

1. Decode inner codewords, get $\hat{e}$.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ \mathbf{0} \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ \mathbf{1} \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}$$

$$\hat{\boldsymbol{e}} = \boxed{0 \ \cdots \ 0 \ \ \mathbf{1} \ \ 0 \ \ \mathbf{1} \ \ 0 \ \ \mathbf{1} \ \ \mathbf{0} \ \ \mathbf{0}}$$

Using the Error Structure for Decoding

Remember: $\boldsymbol{e} = \boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3$

Proposed Decoder

1. Decode inner codewords, get $\hat{e}$.
2. Estimate $\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2$ using $\hat{e}, \boldsymbol{u}_1, \boldsymbol{u}_2$.

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$$\hat{\boldsymbol{r}}_1 = \boxed{0 \ \cdots \ 0 \ \ 0 \ \ 1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0}$$

$$\hat{\boldsymbol{r}}_2 = \boxed{0 \ \cdots \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 1 \ \ 0 \ \ 0 \ \ 0}$$

Using the Error Structure for Decoding

Remember: $\boldsymbol{e} = \boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3$

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1. Decode inner codewords, get $\hat{\boldsymbol{e}}$.
2. Estimate $\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2$ using $\hat{\boldsymbol{e}}, \boldsymbol{u}_1, \boldsymbol{u}_2$.
3. Estimate error $\boldsymbol{e}^* = \boldsymbol{u}_1 \cdot \hat{\boldsymbol{r}}_2 + \boldsymbol{u}_2 \cdot \hat{\boldsymbol{r}}_1$.

$$\boxed{\begin{matrix}0\\0\\1\\1\\1\end{matrix}} \quad \boxed{\begin{matrix}0\\0\\1\\0\\0\end{matrix}} \quad \boxed{\begin{matrix}1\\1\\0\\1\\0\end{matrix}} \quad \boxed{\begin{matrix}1\\1\\1\\1\\1\end{matrix}}$$

$$\begin{aligned}
\hat{\boldsymbol{e}} &= \boxed{0 \ \cdots \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0} \\
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\end{aligned}$$

Using the Error Structure for Decoding

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3. Estimate error $\boldsymbol{e}^* = \boldsymbol{u}_1 \cdot \hat{\boldsymbol{r}}_2 + \boldsymbol{u}_2 \cdot \hat{\boldsymbol{r}}_1$.
4. Decode $\boldsymbol{t}_1 + \boldsymbol{t}_2\boldsymbol{u}_2 - \boldsymbol{e}^* = \boldsymbol{c} + \boldsymbol{e} - \boldsymbol{e}^*$.

$$\boxed{\begin{array}{c}0\\1\\1\\1\\1\end{array}} \quad \boxed{\begin{array}{c}0\\0\\0\\0\\0\end{array}} \quad \boxed{\begin{array}{c}1\\1\\0\\1\\0\end{array}} \quad \boxed{\begin{array}{c}1\\1\\1\\0\\1\end{array}}$$

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1. Decode inner codewords, get $\hat{e}$.
2. Estimate $\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2$ using $\hat{e}, \boldsymbol{u}_1, \boldsymbol{u}_2$.
3. Estimate error $\boldsymbol{e}^* = \boldsymbol{u}_1 \cdot \hat{\boldsymbol{r}}_2 + \boldsymbol{u}_2 \cdot \hat{\boldsymbol{r}}_1$.
4. Decode $\boldsymbol{t}_1 + \boldsymbol{t}_2\boldsymbol{u}_2 - \boldsymbol{e}^* = \boldsymbol{c} + \boldsymbol{e} - \boldsymbol{e}^*$.

$$\boldsymbol{e} - \boldsymbol{e}^* = (\boldsymbol{r}_1 - \hat{\boldsymbol{r}}_1)\boldsymbol{u}_2 + (\boldsymbol{r}_2 - \hat{\boldsymbol{r}}_2)\boldsymbol{u}_1 + \boldsymbol{r}_3$$

$\Rightarrow$ error weight reduced if $\hat{\boldsymbol{r}}_1 \approx \boldsymbol{r}_1$ and $\hat{\boldsymbol{r}}_2 \approx \boldsymbol{r}_2$

$$\boxed{\begin{matrix}0\\1\\1\\1\\1\end{matrix}} \quad \boxed{\begin{matrix}0\\0\\0\\0\\0\end{matrix}} \quad \boxed{\begin{matrix}1\\1\\0\\1\\0\end{matrix}} \quad \boxed{\begin{matrix}1\\1\\1\\0\\1\end{matrix}}$$

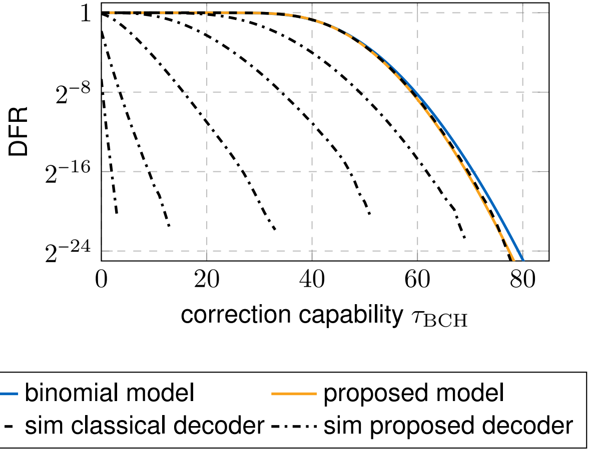
$$\hat{\boldsymbol{e}} = \boxed{0 \;\cdots\; 0 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 0}$$

$$\hat{\boldsymbol{r}}_1 = \boxed{0 \;\cdots\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0}$$

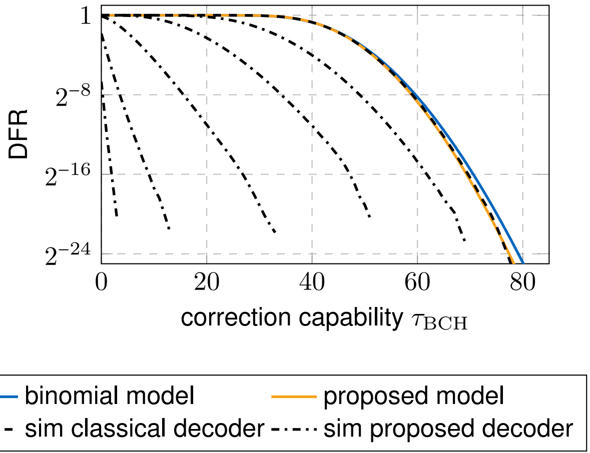
$$\hat{\boldsymbol{r}}_2 = \boxed{0 \;\cdots\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0}$$

$$\boldsymbol{e}^* = \boxed{0 \;\cdots\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0}$$

Decoding Performance Results



Decoding Performance Results



Considerable improvements conceivable ✓

Conclusion

The structure of the HQC error enables

- ☺ tighter DFR estimates
- ☺ improved decoding performance

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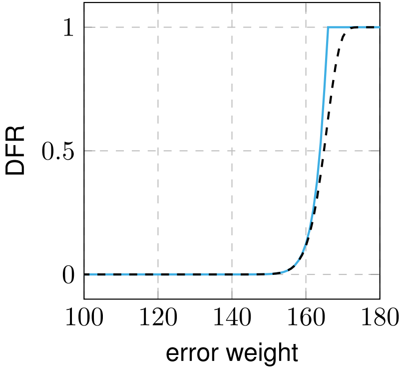
Thank you!

Questions?

A Word about ML Decoding of First-Order RM Codes

- efficient ML decoding:
  Fast Walsh-Hadamard transform
- bound on ML performance



How do we get $\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2$?

Approximate MDPC-like equation: $\hat{e} \approx \boldsymbol{u}_1 \boldsymbol{r}_2 + \boldsymbol{u}_2 \boldsymbol{r}_1$

Correlation Estimate

WLOG, consider $\boldsymbol{r}_1$. Score computation:

$$\sigma_i = \sum_{j \in \mathrm{supp}(\boldsymbol{u}_2)} \mathbb{Z}(\hat{e}_{j+i \mod n}).$$

Threshold decision:

$$\hat{r}_{1,i} = \begin{cases} 1 & \text{if } \sigma_i \geq T, \\ 0 & \text{otherwise,} \end{cases}$$