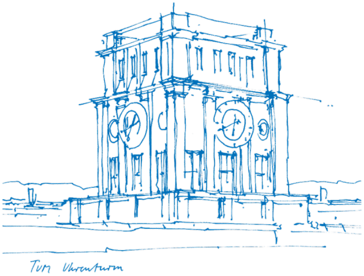
Non-Random Codes
in
Code-Based Cryptography

Sebastian Bitzer
TUM

PICS



Coding and Cryptography (COD)

Professor

Antonia

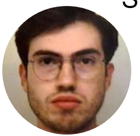
Hugo

Anna

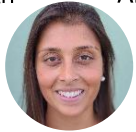
Stefan

Anmoal

me :)



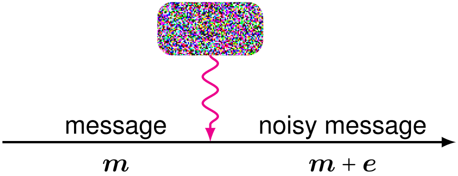
Gökberk



Emma

Sebastian

Channel Coding



$\boldsymbol{m}$ $\quad$ $\boldsymbol{m} + \boldsymbol{e}$

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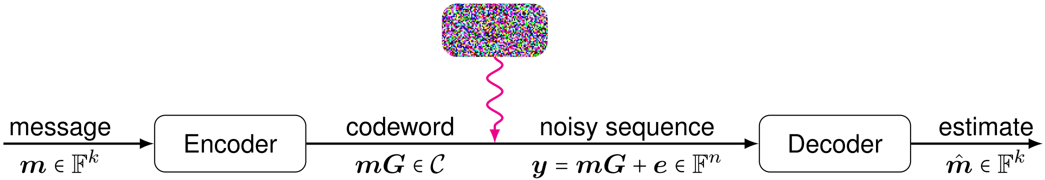

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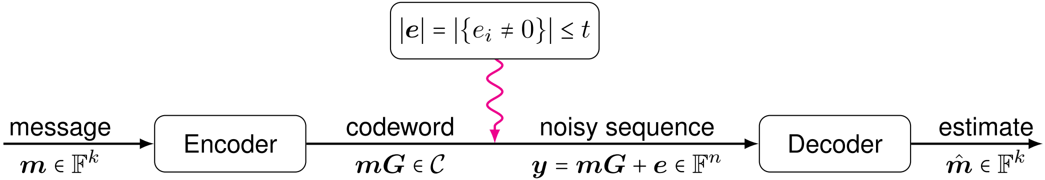

Notations & Definitions

- $\mathcal{C} = \{\boldsymbol{mG} \mid \boldsymbol{m} \in \mathbb{F}^k\} = \{\boldsymbol{c} \mid \boldsymbol{cH}^\top = \boldsymbol{0}\} \subset \mathbb{F}^n$
- Generator matrix $\boldsymbol{G} \in \mathbb{F}^{k \times n}$
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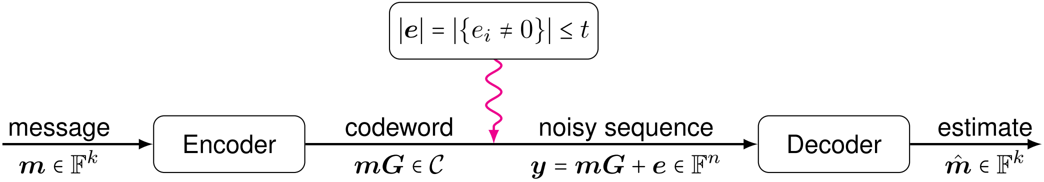

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75 Years of Coding

RS, Goppa, polar, convolutional, . . . codes

➔ structure allows efficient decoding

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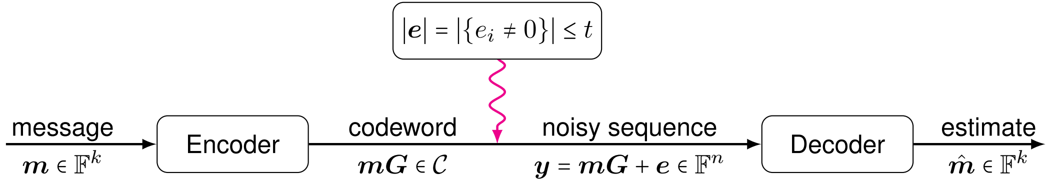

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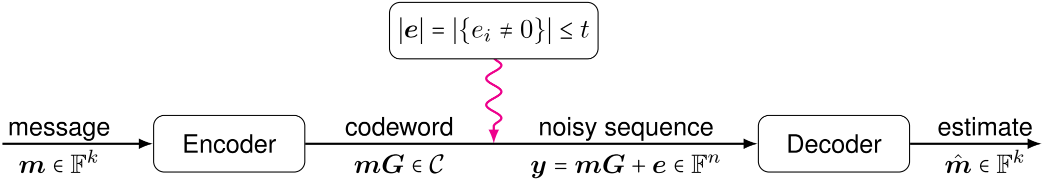

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Code-based Cryptography

Decoding Problem

Given: $\boldsymbol{y} \in \mathbb{F}^n$ and $\boldsymbol{G} \in \mathbb{F}^{k \times n}$
Find: $\boldsymbol{m} \in \mathbb{F}^k$ s.t. $\boldsymbol{y} = \boldsymbol{m}\boldsymbol{G} + \boldsymbol{e}$ with $|\boldsymbol{e}| \leq t$

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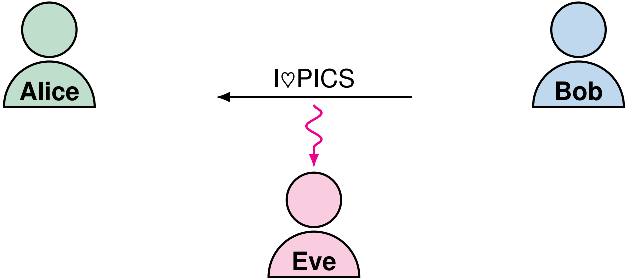

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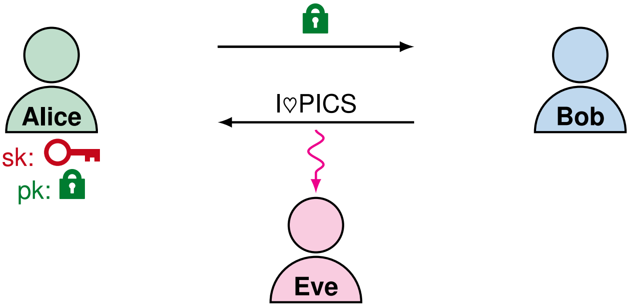

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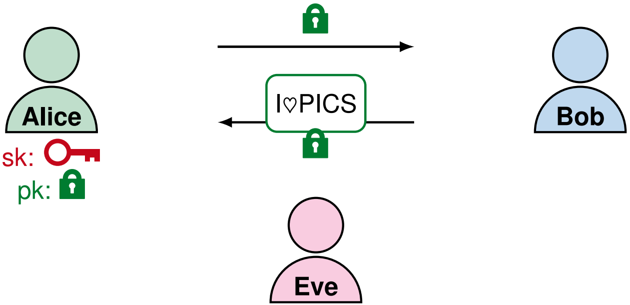

Public-Key Encryption à la McEliece

message $\boldsymbol{m} \in \mathbb{F}^k$

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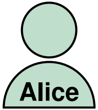

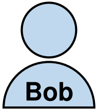

sk: $\mathcal{C}$, $\mathcal{C}$.DEC corrects t errors

message $\boldsymbol{m} \in \mathbb{F}^k$

Public-Key Encryption à la McEliece

Alice

Bob

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message $\boldsymbol{m} \in \mathbb{F}^k$

pk: Generic $\boldsymbol{G} \in \mathbb{F}^{k \times n}$ of $\mathcal{C}$

pk: $\boldsymbol{G}$ →

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pk: $\boldsymbol{G}$ →

← ct: $\boldsymbol{y}$

Bob

message $\boldsymbol{m} \in \mathbb{F}^k$

$\boldsymbol{e} \in \mathbb{F}^n$ with $|\boldsymbol{e}| \leq t$

$\boldsymbol{y} \leftarrow \boldsymbol{m}\boldsymbol{G} + \boldsymbol{e} \in \mathbb{F}^n$

Public-Key Encryption à la McEliece

Alice

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pk: Generic $\boldsymbol{G} \in \mathbb{F}^{k \times n}$ of $\mathcal{C}$

$\hat{\boldsymbol{m}} \leftarrow \mathcal{C}.\textsc{Dec}(\boldsymbol{y})$

Bob

message $\boldsymbol{m} \in \mathbb{F}^k$

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$\boldsymbol{y} \leftarrow \boldsymbol{m}\boldsymbol{G} + \boldsymbol{e} \in \mathbb{F}^n$

Alice → Bob: pk: $\boldsymbol{G}$

Bob → Alice: ct: $\boldsymbol{y}$

Public-Key Encryption à la McEliece

Alice

sk: $\mathcal{C}$, $\mathcal{C}$.DEC corrects t errors

pk: Generic $\boldsymbol{G} \in \mathbb{F}^{k \times n}$ of $\mathcal{C}$

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message $\boldsymbol{m} \in \mathbb{F}^k$

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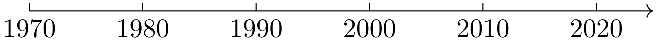
$\boldsymbol{y} \leftarrow \boldsymbol{m}\boldsymbol{G} + \boldsymbol{e} \in \mathbb{F}^n$

Code Requirements

- pk $\boldsymbol{G}$ needs to seem random
- sk $\mathcal{C}$.DEC not revealed by $\boldsymbol{G}$

A Brief History of McEliece

Weger, V., et al. (2022). A survey on code-based cryptography. *Lect. Notes Math.*



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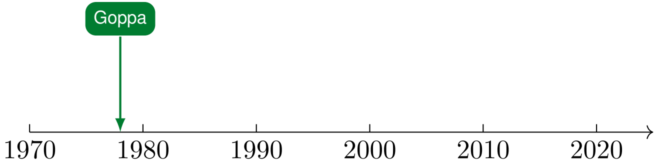

Goppa codes proposed in 1978

A Brief History of McEliece

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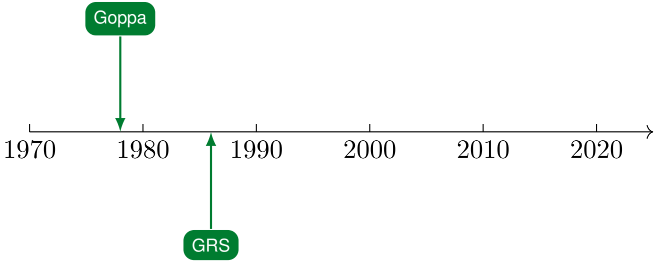

GRS codes proposed in 1986

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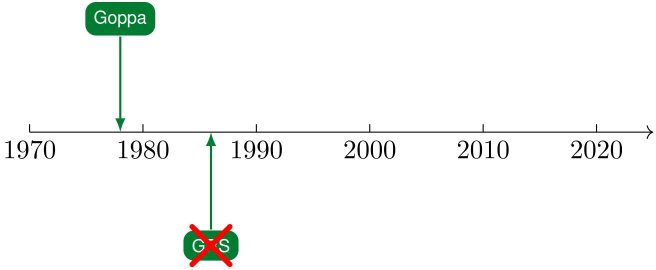

GRS codes proposed in 1986, broken in 1992

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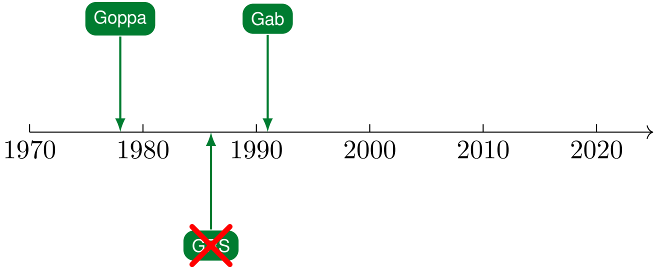

Gabidulin codes proposed in 1991

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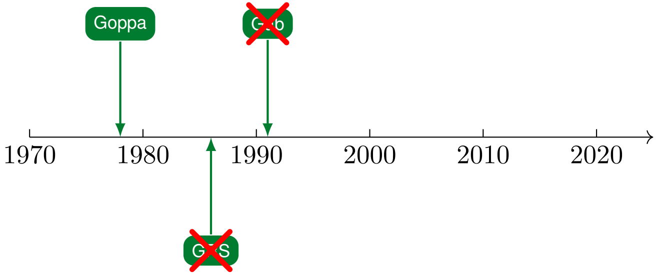

Gabidulin codes proposed in 1991, broken in 2008

A Brief History of McEliece

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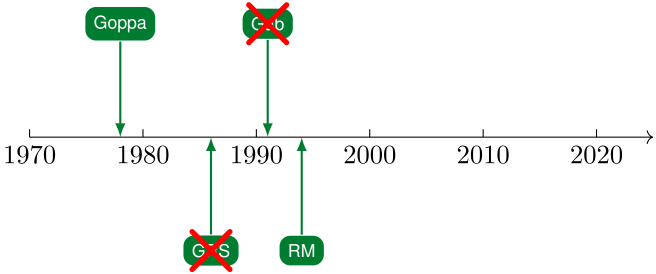

Reed-Muller codes proposed in 1994

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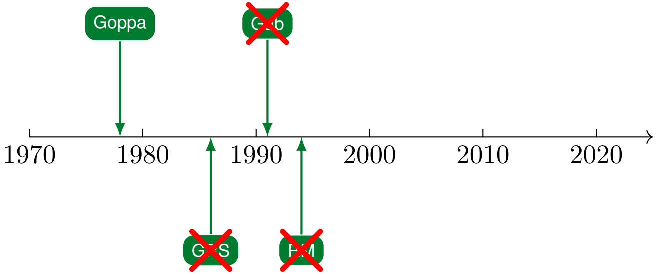
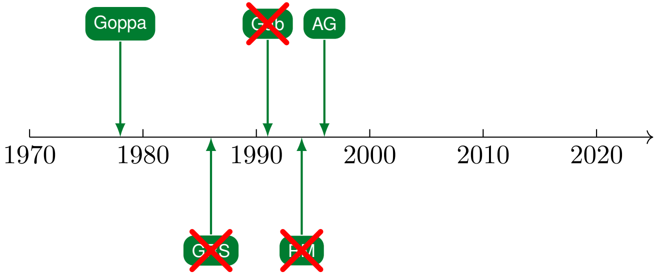


Reed-Muller codes proposed in 1994, broken in 2007

A Brief History of McEliece

Weger, V., et al. (2022).A survey on code-based cryptography. *Lect. Notes Math.*



AG codes proposed in 1996

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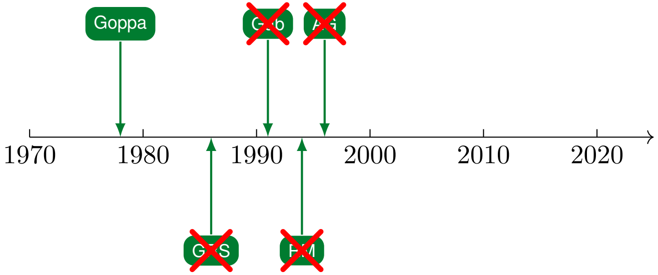

AG codes proposed in 1996, broken in 2014

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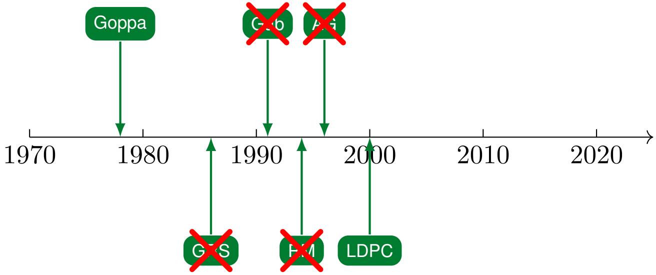

LDPC codes proposed in 2000

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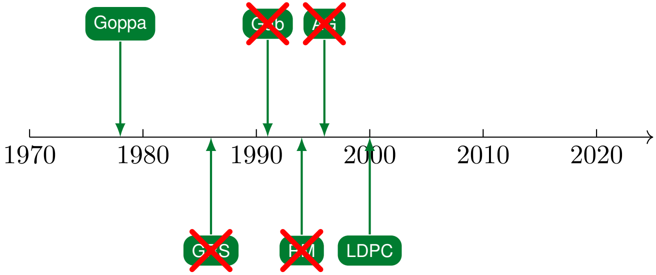

LDPC codes proposed in 2000, modifications required

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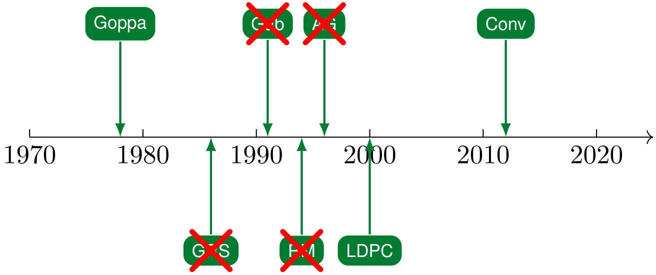

Convolutional codes proposed in 2012

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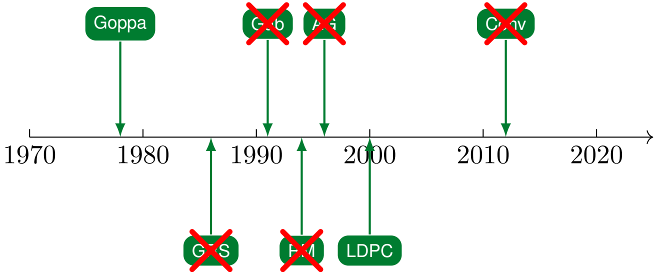

Convolutional codes proposed in 2012, broken in 2013

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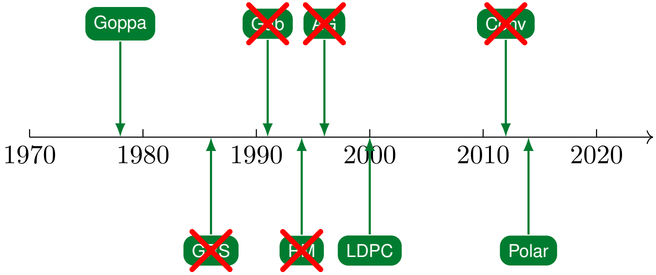

Polar codes proposed in 2014

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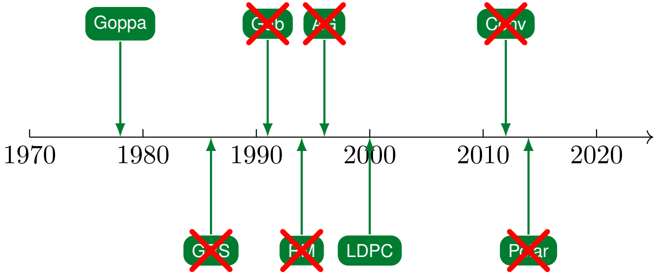

Polar codes proposed in 2014, broken in 2018

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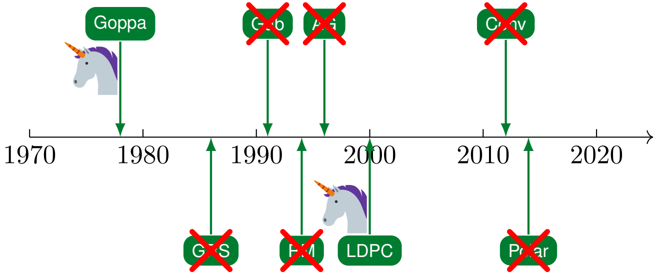

Codes in McEliece

Many have been proposed, almost all insecure

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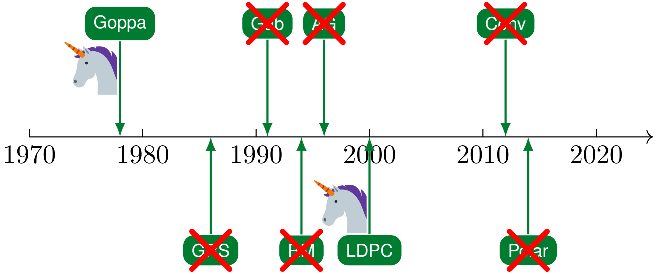

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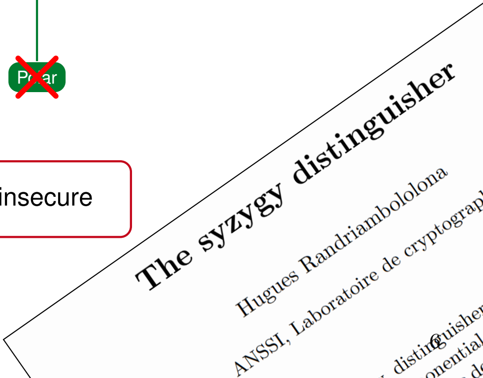
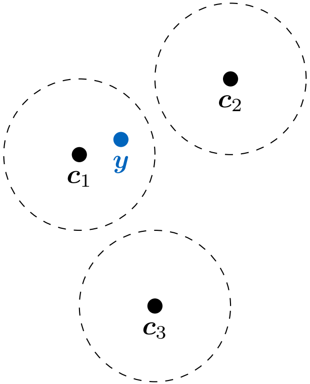

The syzygy distinguisher

Hugues Randriambololona

A Fresh Idea

McEliece's Idea

Efficient decoder but not leaked by $\boldsymbol{G}$



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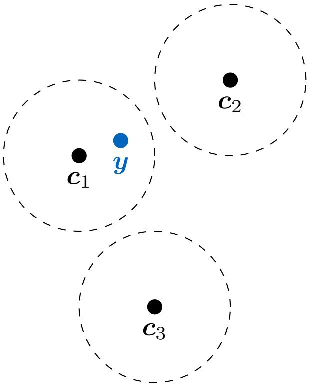

- Aguilar-Melchor, C., et al. (2017). Hamming quasi-cyclic (HQC). *NIST PQC Competition*
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- Structured code (RS+RM)
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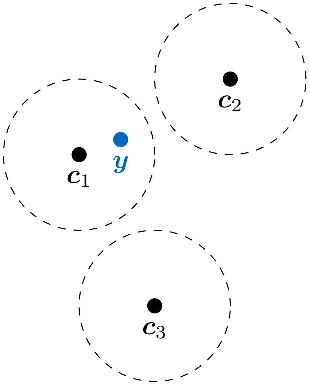

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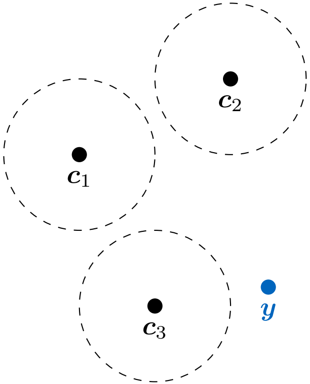

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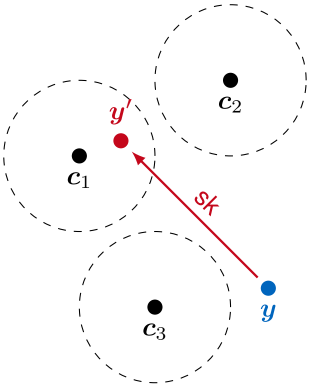

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Put a Ring on It

$$\mathbb{F}^n \qquad\qquad \mathcal{R}_n \coloneqq \mathbb{F}[x]/(x^n - 1)$$

$$\boldsymbol{v} = (v_0, \ldots, v_{n-1}) \qquad\qquad v(x) = \sum_{i=0}^{n-1} v_i x^i$$

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Quasi-Cyclic (QC) SDP

Given: $\boldsymbol{s} \in \mathcal{R}_n$ and $\boldsymbol{h} \in \mathcal{R}_n$

Find: $\boldsymbol{e}_1, \boldsymbol{e}_2$ s.t. $\boldsymbol{e}_1 + \boldsymbol{e}_2\boldsymbol{h} = \boldsymbol{s}$ and $|\boldsymbol{e}_1| + |\boldsymbol{e}_2| \le t$

Put a Ring on It

$$\mathbb{F}^n \qquad\qquad \mathcal{R}_n := \mathbb{F}[x]/(x^n - 1)$$

$$\boldsymbol{v} = (v_0, \ldots, v_{n-1}) \qquad\qquad v(x) = \sum_{i=0}^{n-1} v_i x^i$$

Syndrome Decoding Problem

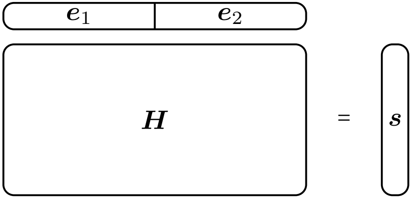
Given: $\boldsymbol{s} \in \mathbb{F}^{n-k}$ and $\boldsymbol{H} \in \mathbb{F}^{(n-k)\times n}$

Find: $\boldsymbol{e} \in \mathbb{F}^n$ s.t. $\boldsymbol{e}\boldsymbol{H}^\top = \boldsymbol{s}$ and $|\boldsymbol{e}| \le t$

Quasi-Cyclic (QC) SDP

Given: $\boldsymbol{s} \in \mathcal{R}_n$ and $\boldsymbol{h} \in \mathcal{R}_n$

Find: $\boldsymbol{e}_1, \boldsymbol{e}_2$ s.t. $\boldsymbol{e}_1 + \boldsymbol{e}_2\boldsymbol{h} = \boldsymbol{s}$ and $|\boldsymbol{e}_1| + |\boldsymbol{e}_2| \le t$



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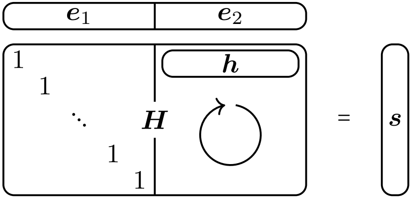
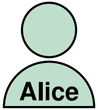
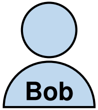


HQC in a Nutshell



$$\mathcal{R}_n = \mathbb{F}[x]/(x^n - 1)$$



message $\boldsymbol{m} \in \mathbb{F}^k$

HQC in a Nutshell

$\mathcal{R}_n = \mathbb{F}[x]/(x^n - 1)$

Alice

Bob

$\boldsymbol{h} \in \mathcal{R}_n$

message $\boldsymbol{m} \in \mathbb{F}^k$

sk: $\boldsymbol{u}_1, \boldsymbol{u}_2 \in \mathcal{R}_n$ of wt w_u

pk: $\boldsymbol{s} \leftarrow \boldsymbol{u}_1 + \boldsymbol{h}\boldsymbol{u}_2$

pk: $(\boldsymbol{h}, \boldsymbol{s})$ →

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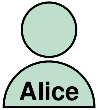

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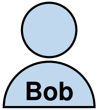

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$\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \in \mathcal{R}_n$ of wt w_r

$\boldsymbol{y}_1 \leftarrow \boldsymbol{m}\boldsymbol{G} + \boldsymbol{s}\boldsymbol{r}_2 + \boldsymbol{r}_3$

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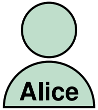

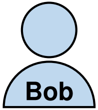

Alice:

$\boldsymbol{h} \in \mathcal{R}_n$

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Alice → Bob: pk: $(\boldsymbol{h}, \boldsymbol{s})$

Bob:

message $\boldsymbol{m} \in \mathbb{F}^k$

$\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \in \mathcal{R}_n$ of wt w_r

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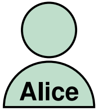
Bob → Alice: ct: $(\boldsymbol{y}_1, \boldsymbol{y}_2)$

Alice:

$\hat{\boldsymbol{m}} \leftarrow \mathcal{C}.\textsc{Dec}(\boldsymbol{y}_1 - \boldsymbol{y}_2\boldsymbol{u}_2)$

HQC in a Nutshell



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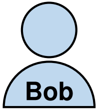

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pk: $(\boldsymbol{h}, \boldsymbol{s})$ →

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← ct: $(\boldsymbol{y}_1, \boldsymbol{y}_2)$

$\boldsymbol{y}_2 \leftarrow \boldsymbol{r}_1 + \boldsymbol{h}\boldsymbol{r}_2$

$\mathcal{C}$ needs to decode $\boldsymbol{y}_1 - \boldsymbol{y}_2\boldsymbol{u}_2 = \boldsymbol{c} + \underbrace{\boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3}_{\text{error } \boldsymbol{e}}$

Decryption Failure Is Not an Option

Security Issues

- IND-CCA security
- Reaction attacks

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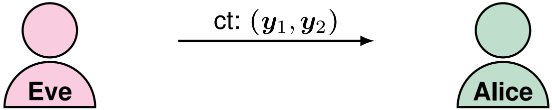

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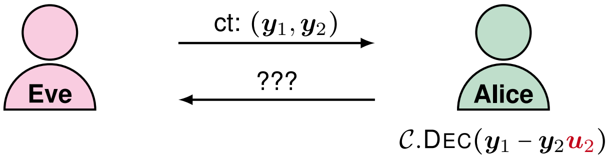

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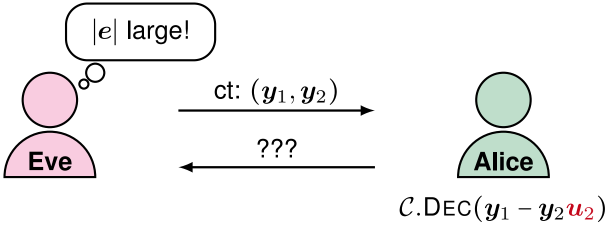

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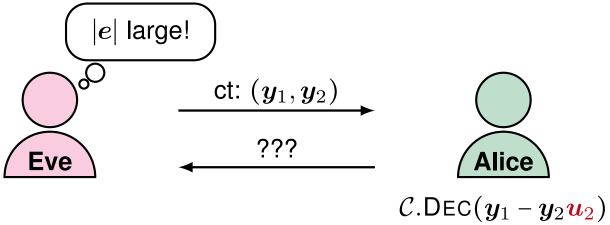

Guo, Q., & Johansson, T. (2020). A new decryption failure attack against HQC.

→ DFR needs to be $\leq 2^{-128}$

A First Look at the Error

$P(|\boldsymbol{e}| = w)$ difficult for $\boldsymbol{e} = \boldsymbol{u}_1\boldsymbol{r}_2 + \boldsymbol{u}_2\boldsymbol{r}_1 + \boldsymbol{r}_3$

$\rho = P(e_i = 1)$ simple

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Under the independence assumption,

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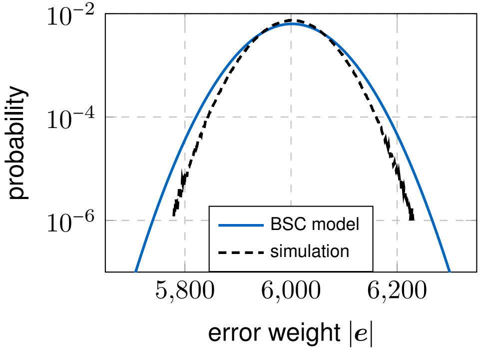

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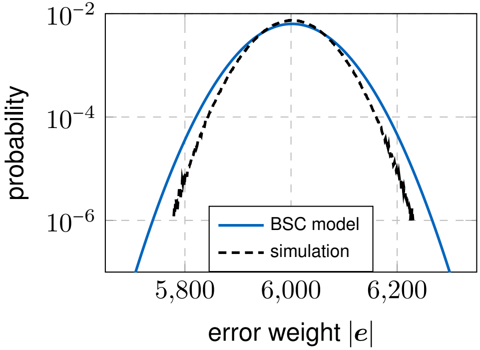
Heuristic for weight after multiplication



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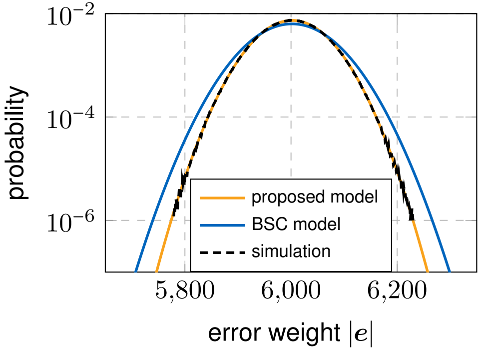
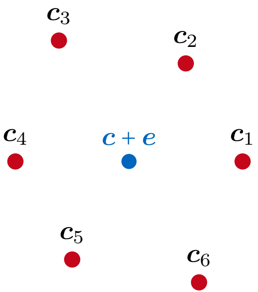


Beyond the BSC

plausible for BSC



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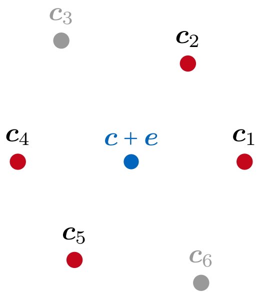

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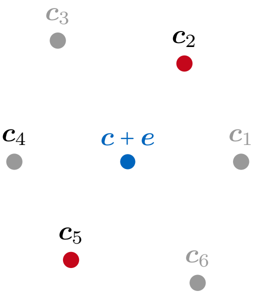

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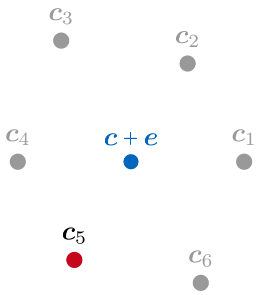

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with known $\boldsymbol{u}_1, \boldsymbol{u}_2$

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- ☺ No DFR, no heuristics
- ☺ better parameters
- (!) explicit code needed
- (!) efficient decoder needed

Conclusion

Non-random codes in code-based cryptography:

- ☺ McEliece has strong code requirements
- ☺ HQC allows public decoder
- ☺ Error structure of HQC

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