Ciphertext Compression for HQC

Sebastian Bitzer   Emma Munisamy   Bharath Purtipli   Stefan Ritterhoff
Antonia Wachter-Zeh

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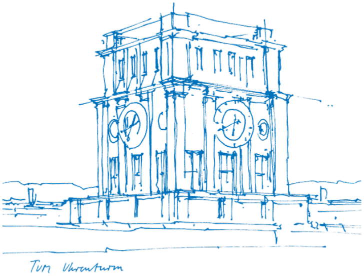

NIST's favourite KEMs

HQC	KYBER

Regev, On Lattices, Learning with Errors, Random Linear Codes, and Cryptography

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This talk:

- → Hamming-metric compression
- → Reduce ciphertext size

HQC in a Nutshell

KeyGen

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\begin{aligned}
\boldsymbol{e}_1, \boldsymbol{e}_2 &\leftarrow \mathcal{S}_w^n \\
\boldsymbol{h} &\leftarrow \mathbb{F}_2[x]/(x^n - 1) \\
\boldsymbol{s} &= \boldsymbol{h} \cdot \boldsymbol{e}_1 + \boldsymbol{e}_2
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`pk`: $(\boldsymbol{h}, \boldsymbol{s})$

Encrypt

$$\begin{aligned} \boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 &\leftarrow \mathcal{S}_{w'}^n \\ \boldsymbol{y}_1 &= \boldsymbol{s} \cdot \boldsymbol{r}_2 + \boldsymbol{r}_3 + \mathcal{C}.\mathsf{enc}(\boldsymbol{m}) \\ \boldsymbol{y}_2 &= \boldsymbol{r}_1 + \boldsymbol{h} \cdot \boldsymbol{r}_2 \end{aligned}$$

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sk: $\boldsymbol{e}_1$

ct: $(\boldsymbol{y}_1, \boldsymbol{y}_2)$

Decrypt

$$\hat{\boldsymbol{m}} = \mathcal{C}.\mathtt{dec}(\boldsymbol{y}_1 - \boldsymbol{e}_1 \cdot \boldsymbol{y}_2)$$

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Decrypt

$$\hat{\mathbf{m}} = \mathbf{m} + \mathcal{C}.\mathsf{dec}(\mathbf{e}_{\mathrm{HQC}})$$

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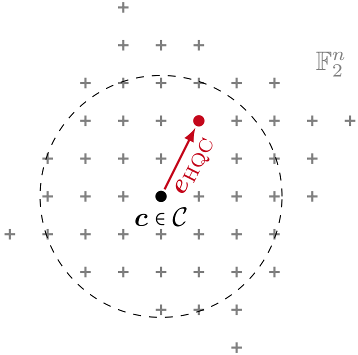
Less Bits, More Noise

$$\lfloor \cdot \rceil_{\mathcal{Q}} : \quad \mathbb{F}_2^n \to \mathcal{Q}$$

$$\boldsymbol{v} \mapsto \lfloor \boldsymbol{v} \rceil_{\mathcal{Q}} = \boldsymbol{v} + \{\boldsymbol{v}\}_{\mathcal{Q}}$$

$$n\,\text{bit} \to \rho n\,\text{bit}, \quad \rho < 1$$



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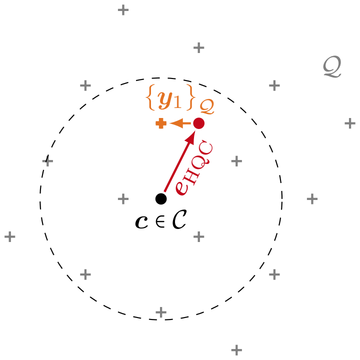

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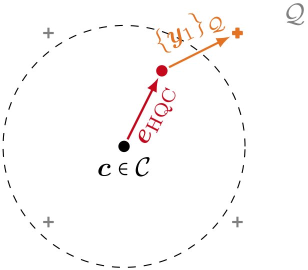

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Claude E. Shannon, A Mathematical Theory of Communication

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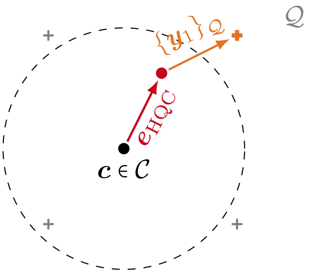
Rate-Distortion Bound (RDB)

Compression ρ implies average distortion

$$\mathbb{E}_{\boldsymbol{v}}\left[\tfrac{1}{n}\mathrm{wt}_{\mathrm{H}}(\{\boldsymbol{v}\}_{\mathcal{Q}})\right] \geq H^{-1}(1-\rho)$$



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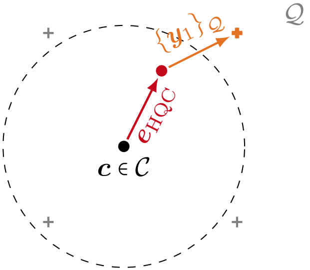

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Trade-off: ct size vs. pk size

Polar Codes are Good Quantizers

Korada, Urbanke, Polar Codes are Optimal for Lossy Source Coding

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➔ Successive Cancellation (SC) decoding:
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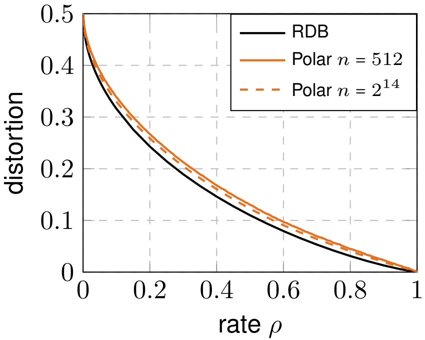

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Korada, Urbanke, Polar Codes are Optimal for Lossy Source Coding

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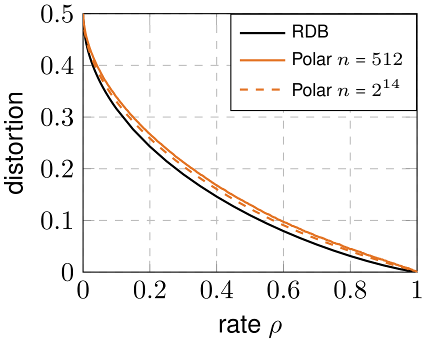
- Distortion statistically close to BSC



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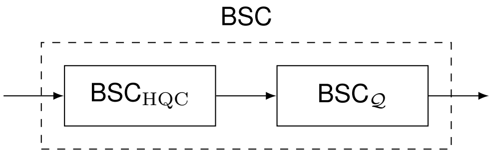
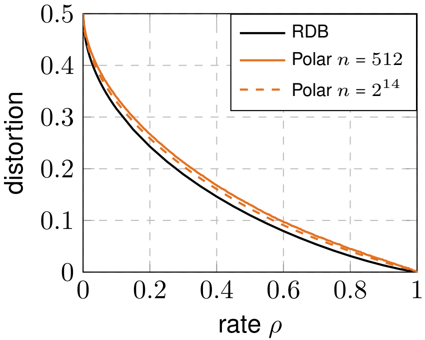
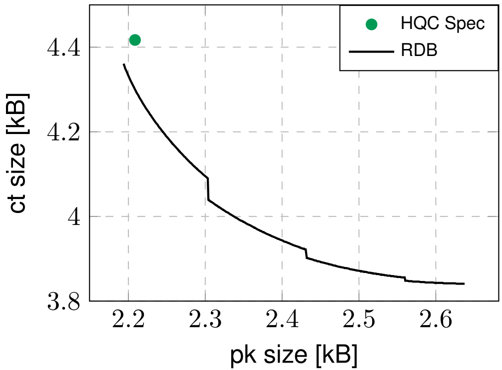




Polarizing Results



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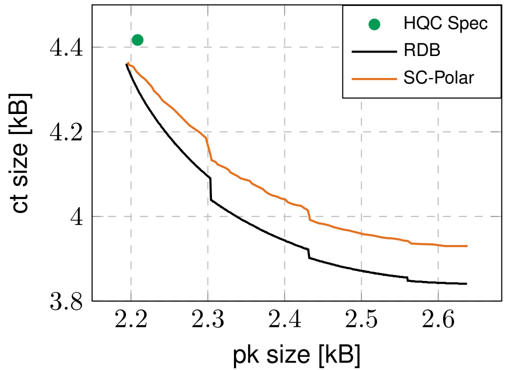

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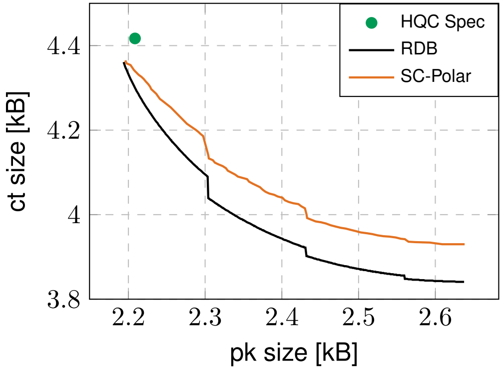

Optimize compression:

- Better code, better decoder
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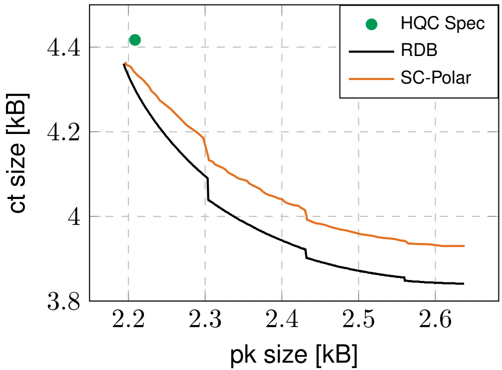

Optimize compression:

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- (!) More complex

Simplify compression:

- ☺ Low cost, rigorous analysis
- (!) Suboptimal compression

Keep It Short & Simple

Claude E. Shannon, Coding Theorems for a Discrete Source with a Fidelity Criterion

Direct Product $\mathcal{Q} \times \cdots \times \mathcal{Q}$

- ➔ $\mathcal{V}(\mathcal{Q}) = \{\boldsymbol{v} \in \mathbb{F}_2^n : \lfloor \boldsymbol{v} \rceil_{\mathcal{Q}} = \mathbf{0}\}$
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Golay Code

- $n = 23,\ \rho = \frac{12}{23}$
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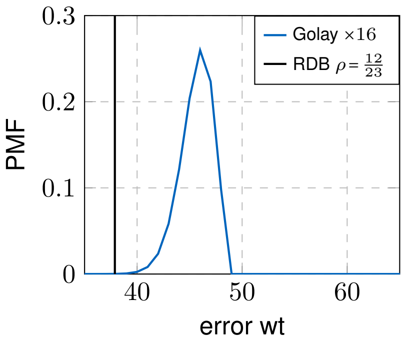

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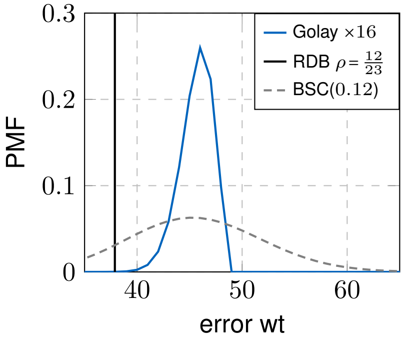
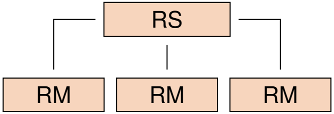


Aligning Compression and Error Correction



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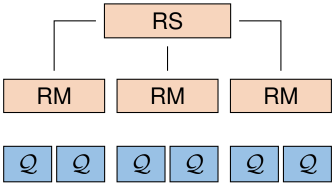

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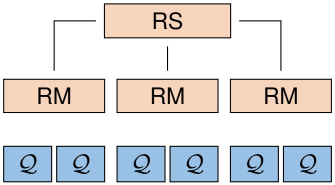

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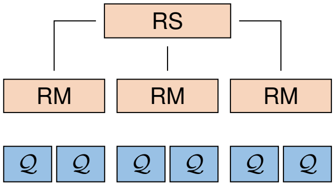

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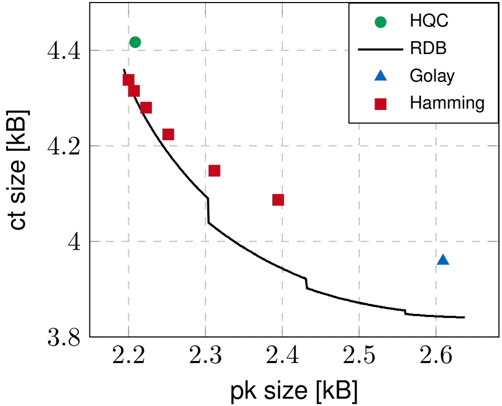

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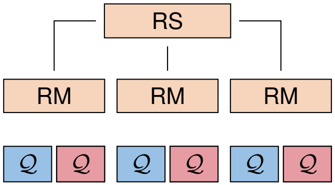

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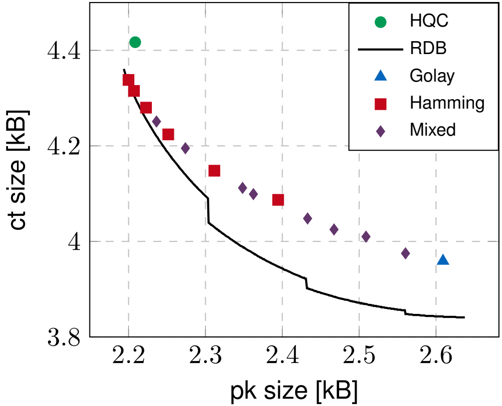

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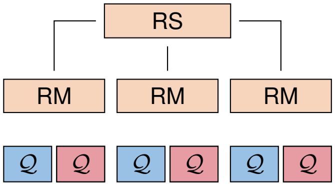

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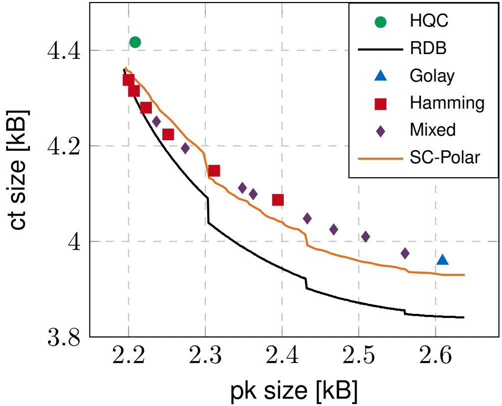

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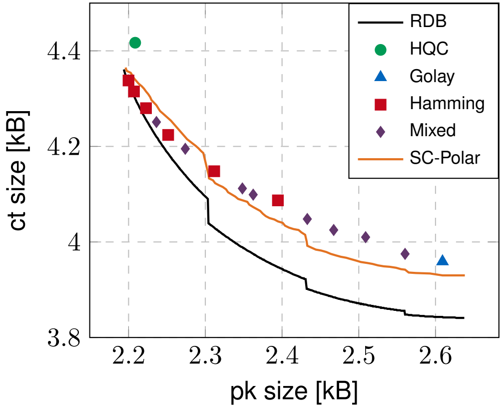

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Questions?